# Geometry, Mesopotamian

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Already in late pre-literate times the ground plans of prestige buildings were laid out with string. In proto-Elamite Tepe Yahya, buildings were planned using a modular unit; the definition of standard bricks in terms of the length unit shows the same to be the case in later Mesopotamia. In both cases, and in earlier as well as prestige building, right angles are fundamental. These, in the guise of squares and rectangles, are also important in the geometric sensibility expressed in proto-literate and later tablet formats and decoration.

Calculating geometry goes back to the proto-literate period, where area units were normalized and henceforth based on the square of the length unit. From that time we also have evidence for the calculation of almost-rectangular areas by means of the "surveyors' formula", average length times average width. Field plans from later periods still subdivide into approximate rectangles, right triangles and right trapezia, having no interest in rendering non-right angles exactly; heights are transformed into dividing lines. Even in late Babylonian astronomy, the measurement of heavenly distances does not correspond to a general angle concept.

Nonetheless, similarity (probably understood as "same shape") was fundamental. Tables of technical geometrical constants (e.g., for the diagonal of a square and the area of a circle) presuppose multiplication by a linear extension (for one-dimensional magnitudes like the diagonal) or its square (for areas). These linear extensions are normally external, like the perimeter of the circle, which allows direct measurement. The ratio between the circular perimeter and diameter was supposed to be 3:1. From Old Babylonian times onward, the "Pythagorean rule" was used.

Prismatic volumes were determined as base time height, truncated pyramids and cones mostly as average base times height.

"Supra-utilitarian" geometry (see MATHEMATICS, MESOPOTAMIAN) was interested in figures divided by parallel transversals. Already a Sargonic tablet shows knowledge of the rule for bisection of a trapezium. There was also interest in subdividing squares by insertion of smaller squares, diagonals and circles. It always aimed at determining a number, interest in explicit demonstration was absent.

A particular branch of supra-utilitarian geometry is the so-called "algebra", a technique based on the transformation of squares and rectangles with measurable sides by means of cut-and-paste procedures and proportional stretching in one direction. If $\ell$ and $w$ are the sides of a rectangle and $A$ its area, the former technique allows the finding of $\ell$ and $w$ from $A$ and $\ell \pm w$; if $s$ is the side of a square with area $\square(s)$, the latter allows transformation of problems $\alpha\square(s)\pm\beta s = \gamma$ into $\square(\alpha s)\pm\beta(\alpha s) = \alpha\gamma$ ($\alpha$, $\beta$ and $\gamma$ being numbers, and $\square(s)$ the area of a square with side $s$). This developed into a sophisticated technique in the Old Babylonian school solving all kinds of second-degree problems, plausibly starting from a small set of surveyors' geometrical riddles but using the linear magnitudes as *representatives* for prices, numbers, numbers of men or days, and even areas or volumes. It served no practical purpose, only as display of virtuosity; it disappeared with the Old Babylonian school. Some of the original riddles return in Late Babylonian times.